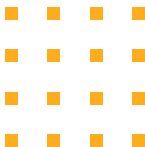




Class 4: Counting and Probability



Warm-up Problem 1

Calculate the following values without a calculator or pencil/paper:

(a) $25 \times 23 \times 4$

(b) $199 + 365$

(a) $25 \times 23 \times 4 = 100 \times 23$

$$= 2300$$

(b) $199 + 365 = 200 + 365 - 1$

$$= 565 - 1$$

$$= 564$$

Warm-up Problem 2

Calculate the following values without a calculator or pencil/paper:

(a) $28 + 173 + 72$

(b) $364 - (54 + 64)$

$$(a) \ 28 + 173 + 72 = (28 + 72) + 173$$

$$= 100 + 173$$

$$= 173$$

$$(b) \ 364 - (54 + 64) = 364 - 54 - 64$$

$$= (364 - 64) - 54$$

$$= 300 - 54$$

$$= 246$$

Homework Take-Up – Question 1

Given a number sequence 3, 7, 6, 6, 2, 3, 7, 6, 6, 2, ...

What is the sum of the first 101 numbers?

The numbers are repeating after every 5 numbers – 3, 7, 6, 6, 2

$$101 \div 5 = 20 \text{ R}1$$

$$3 + 7 + 6 + 6 + 2 = 24 \quad \rightarrow \text{The sum of one cycle is 24}$$

$$20 \times 24 = 480 \quad \rightarrow \text{The sum of 20 cycles is 480}$$

Sum of the first 100 numbers = 480

The 101st number is the first number of the sequence, 3.

$$\text{Sum of first 101 numbers} = 480 + 3 = 483$$

\therefore The sum of the first 101 numbers is 483.

Homework Take-Up – Question 2

Harry goes outside for a run every 3 days and goes to the gym every 8 days. If he went for a run and went to the gym today, in how many days will be the next day he runs and goes to the gym on the same day?

We need to find the Lowest Common Multiple of 3 and 8, which will be the next time he runs and goes to the gym on the same day.

Prime factorize 3 and 8:

$$3 = 3$$

$$8 = 2^3$$

$$\begin{aligned}\text{lcm}(3, 8) &= 2^3 \times 3 \\ &= 24\end{aligned}$$

\therefore He will run and go to the gym in 24 days.

Probability

Probability is the likelihood of an event happening. For example, the probability of the temperature being warm today is very high, but the probability that I can win the lottery is very low.

$$\text{Probability} = \frac{\text{Desired Outcome}}{\text{Total Outcomes}}$$

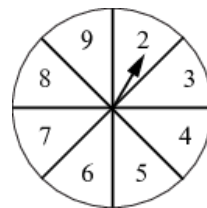
$$\text{Probability} = \frac{\text{Desired Outcome}}{\text{Total Outcomes}}$$

Problem

A circular spinner is divided into 8 equal sections, as shown. When the arrow is spun once, what is the probability that the arrow stops at an odd number?

The odd numbers on the spinner are 3, 5, 7, and 9.

Since there are 8 sections in total, the probability of getting a 3, 5, 7, or 9 is $4/8 = 1/2$.



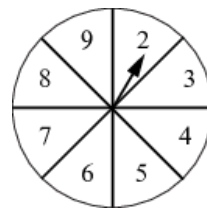
$$\text{Probability} = \frac{\text{Desired Outcome}}{\text{Total Outcomes}}$$

Problem

A circular spinner is divided into 8 equal sections, as shown. When the arrow is spun once, what is the probability that the arrow stops in a section containing a prime number that is odd? (Source: Variation from Gauss 7)

The prime numbers that are odd on this spinner are 3, 5, and 7.

Since there are 8 sections in total, the probability of getting a 3, 5, or 7 is $\frac{3}{8}$.



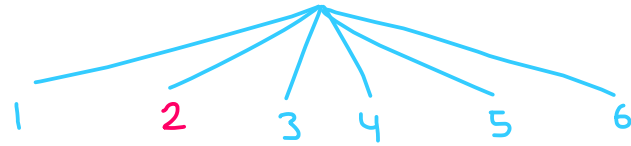
$$\text{Probability} = \frac{\text{Desired Outcome}}{\text{Total Outcomes}}$$

Problem

What is the probability of rolling a 2 on a die?

Since there are 6 faces on a die, and each face has an equal probability, the probability of rolling each one is $1/6$.

\therefore The probability of rolling a 2 is $1/6$.



$$\text{Probability} = \frac{\text{Desired Outcome}}{\text{Total Outcomes}}$$

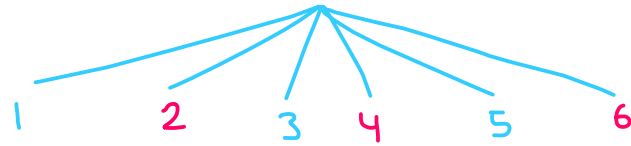
Problem

What is the probability of rolling an even number on a die?

There are three even faces on a die – 2, 4, and 6.

There are 6 faces in total and each one has an equal chance of being rolled, so the probability is $3/6 = 1/2$.

∴ The probability of rolling an even number is $1/2$.



Problem

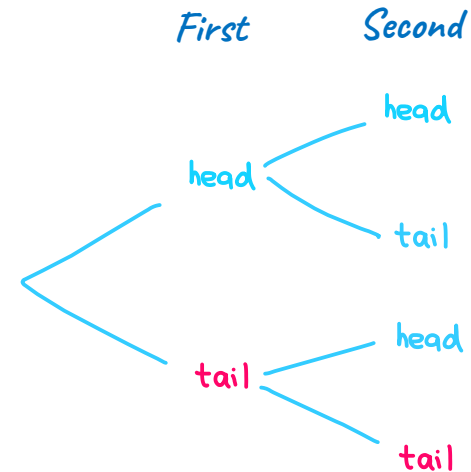
What is the probability of flipping 2 coins and getting 2 tails in a row?

First flip: $\frac{1}{2}$ chance of tails

Second flip: $\frac{1}{2}$ chance of tails

Total chance of ALL tails = $\frac{1}{2} \times \frac{1}{2}$
= $\frac{1}{4}$

\therefore The probability of rolling two of tails is $\frac{1}{4}$.



Problem

What is the probability of flipping 3 coins and getting 3 heads in a row?

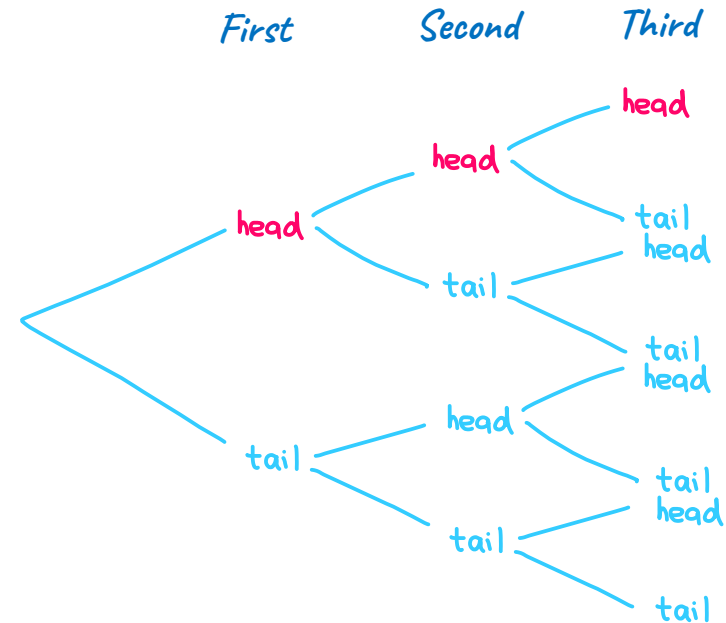
First flip: $\frac{1}{2}$ chance of heads

Second flip: $\frac{1}{2}$ chance of heads

Third flip: $\frac{1}{2}$ chance of heads

Total chance of ALL heads = $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$
= $\frac{1}{8}$

\therefore The probability of rolling three heads is $\frac{1}{8}$.

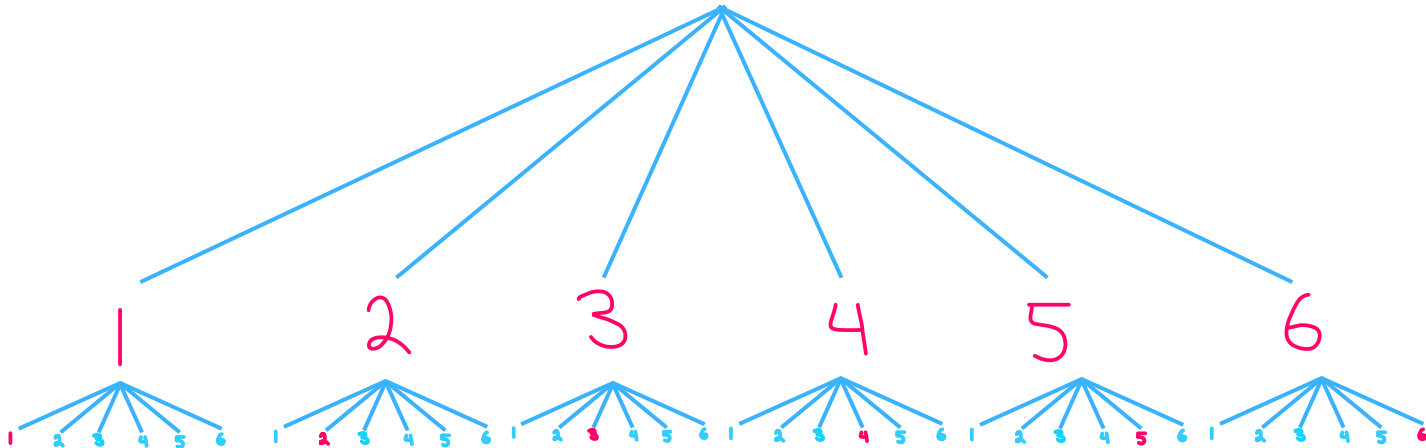


Problem

What is the probability of rolling two dice and getting the same number on both?

If you roll a number on the first die, there is a $1/6$ chance that you will roll that number on your second die.

\therefore The probability of rolling two of the same numbers is $1/6$.



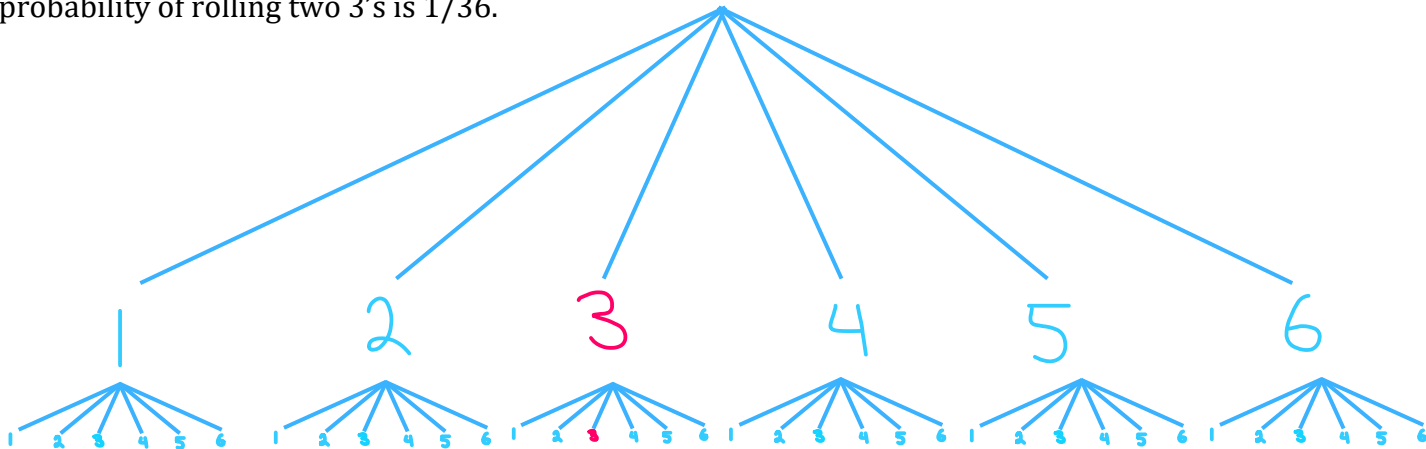
Problem

What is the probability of rolling two dice and 3 on both?

There is a $1/6$ chance of rolling 3 on your first die, and another $1/6$ chance of rolling 3 on your second die.

Total Probability = $1/6 \times 1/6 = 1/36$

\therefore The probability of rolling two 3's is $1/36$.



Combinations and Permutations

Combinations and permutations both describe the ways that things can be selected from a set.

Combinations are when order doesn't matter.

Permutations are when order matters.

For example, choosing 3 foods to eat from a list of 10 foods is a **combination**, since it doesn't matter which order you pick them in.

However, if you rank your top 3 foods from a list of 10 foods, that is a **permutation**, since the order matters.

Exercise

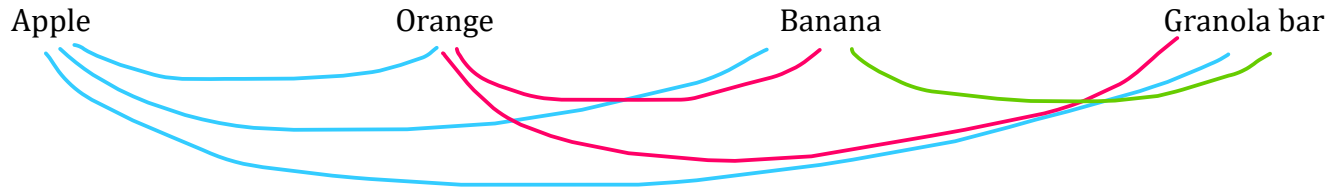
Are the following combinations or permutations?

- (a) Picking a team of 5 people from a group of 20.
- (b) Choosing a President, Vice-President, and Secretary from a group of 100 people for a council.
- (c) Arranging 10 people into a line.
- (d) Ways to choose 2 snacks from a store.

- (a) Combination – the order of the people doesn't matter, since there just needs to be 5 people
- (b) Permutation – the order of the people matters – each person will get a different position in the council
- (c) Permutation – the order of the people matters (it's a line)
- (d) Combination – the order of the snacks doesn't matter, you only have to pick 2 snacks

Problem

Jacob chooses two different items for a snack. His choices are an apple, an orange, a banana, and a granola bar. How many different pairs of snacks could he choose?

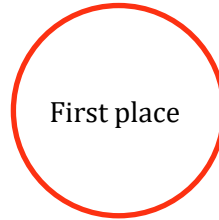


∴ There are 6 different pairs that he can choose.

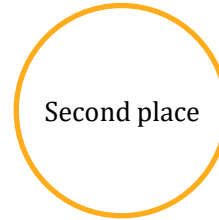
Problem

Harry, Ron, and Neville are having a race on their broomsticks. If there are no ties, in how many different possible orders can they finish? (Source: Gauss 7)

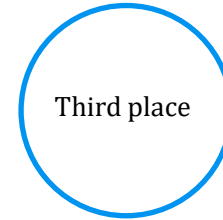
Harry – Ron – Neville
Harry – Neville – Ron
Ron – Harry – Neville
Ron – Neville – Harry
Neville – Harry – Ron
Neville – Ron – Harry



3 choices



2 choices



1 choice (the remaining person)

$3 \times 2 \times 1 = 6$ ways to arrange them

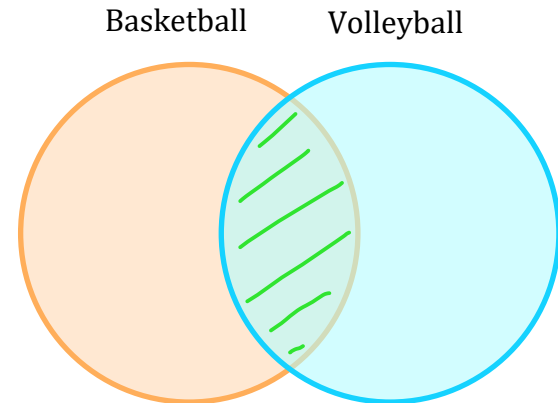
∴ There are 6 different orders they can finish.

Problem

A survey was conducted to all grade 6 students at some school. There are 120 people who like to play basketball, 100 people who like to play volleyball, and 50 people who like to play both. If every student chose at least one of these sports, how many grade 6 students are there at this school?

$$\begin{aligned}\text{Total number of students} &= (\text{Students who like basketball}) + (\text{Students who like volleyball}) - (\text{Students who like both}) \\ &= 120 + 100 - 50 \\ &= 170\end{aligned}$$

∴ There are 170 students.

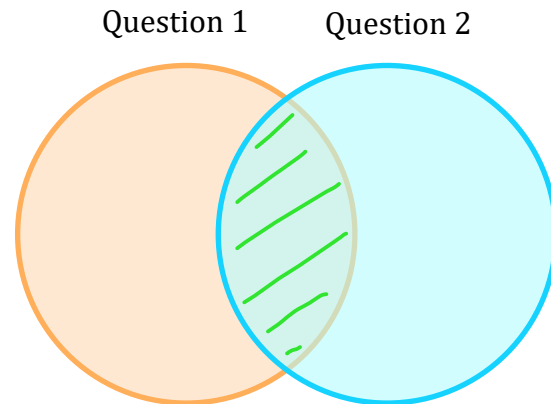


Problem

On a math test, 15 students got the first question correct, 10 students got the second question correct, and 7 students got both questions correct. Everyone got at least one question correct. How many students are there?

$$\begin{aligned}\text{Total number of students} &= (\text{First question correct}) + (\text{Second question correct}) - (\text{Both questions correct}) \\ &= 15 + 10 - 7 \\ &= 18\end{aligned}$$

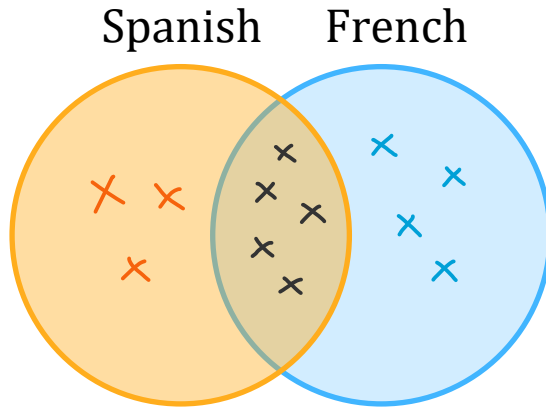
\therefore There are 18 students.



Problem

There are 12 people who are taking at least one foreign language class. The school offers only Spanish and French as its foreign language classes. 8 are taking Spanish and 5 are taking both languages. How many people are taking French?

Solution 1:



\therefore There are 9 people taking French.

Problem

There are 12 people who are taking at least one foreign language class. The school offers only Spanish and French as its foreign language classes. 8 are taking Spanish and 5 are taking both languages. How many players are taking French?

Solution 2:

Total number of people = (People taking Spanish) + (People taking French) - (People taking both)

$$8 + (\text{People taking French}) - 5 = 12$$

$$(\text{People taking French}) + 3 = 12$$

$$\text{People taking French} = 9$$

∴ There are 9 people taking French.

